3th International Conference on Public Communication of Science and Technology (PCST), Montreal,Canada,10-13 April,1994

# CAN MATHEMATICS PLAY A CULTURAL ROLE?

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*Mathematica est scientia intellectus*
Tommaso Campanella (1568-1639)

## Introduction

In an essay significantly entitled "Mathematics in Western Culture", the mathematician Morris Kline wrote:[1] "The object of this book is to advance the thesis that mathematics has been a major cultural force in Western civilization". And he added:

"Fewer people seem to be aware that mathematics carries the main burden of scientific reasoning and is the core of the major theories of physical science. It is even less widely known that mathematics has determined the direction and content of much philosophic thought, has destroyed and rebuilt religious doctrines, has supplied substance to economic and political theories, has fashioned major painting, musical, architectural, and literary styles, has fathered our logic, and has furnished the best answers we have to fundamental questions about the nature of man and his universe. Finally, as an incomparably fine human achievement, mathematics offers satisfactions and aesthetic values at least equal to those offered by any other branch of our culture."

In spite of this, there are very few educated people who consider mathematics to be a subject of intellectual interest. And worse still, as Kline points out:[2] "a subject that is basic, vital and elevating is neglected and even scorned by otherwise highly educated people. Indeed, ignorance of mathematics has attained the status of a social grace."

These words were written almost forty years ago. Has any change taken place in the meantime? Recently the famous French mathematician Jean Dieudonné suggested, in his vast work entitled "Pour l'honneur de l'esprit humain":[3] "Within the framework of human activity, the situation occupied by mathematics is paradoxical." In fact, most people in industrialized countries recognize that mathematics is a fundamental discipline touching nearly all branches of science and technology; in addition, quite a widespread opinion is that the mere fact of having a reasonable knowledge of mathematics opens the way to more and more work opportunities. This aspect of mathematics has been recognized, though only in part, for some time. Proof is provided by the fact that Ulrich, the hero of "The Man Without Quality" says that[4]: "there is no need to examine the subject in greater depth, since almost everyone (the first edition of the book was in 1930) is well aware that mathematics has entered into all applications of life, like a demon."

Except that he goes on to say that although not everyone believes in the story of the devil to whom one can sell one's soul, "those who know about the soul – in other words, priests, historians, artists – mantain that the soul has been ruined by mathematics, and that mathematics is at the root of a perfidious system of reasoning that does, in fact, make Mankind the master of the world, but also the slave of machines." Furthermore, according to this argument, the collapse of European culture has taken place because "Mankind no longer has a place in his heart for faith, love, innocence or goodness."

But who thought, and probably still thinks, like that? Musil ironically pointed out that people who have such a low opinion of mathematics must have been bad students in their school days, and that this accounts for the envy that motivates them! By contrast, this attitude helped Ulrich to increase his "more human than scientific" love of science. "He loved mathematics on account of those who could not stand it".

Many people are averse to mathematics, others have no reaction through lack of interest, while the largest number do not know what it is all about. This makes it even more difficult to answer the question as to the cultural role that mathematics has had in the past, and can have today. This is why Dieudonné, talking to the layman, first parts the basic question:

"What is mathematics?" or its extension "What does a mathematician do?" Answering these questions is not an easy matter if, as Dieudonné points out, the interlocutor has not studied mathematics for at least two years at university level. If this is not the case, it is highly likely that the answers will be absurd. And this is true not only for those who are not particularly well-educated, but also for "eminent figures from other scientific disciplines" who often have weird ideas about what mathematicians do.

Not only do most people not have the faintest idea about what mathematicians do, but also one of the most widespread ideas (and one of the most mistaken, as Dieudonné points out) is that in mathematics there is nothing new to be discovered, and mathematicians are limited to teaching what has been inherited from the past. This of course, helps to fuel the talk *ad nauseam* in the mass media of progress in all other branches of science. To summarize the most widely held opinions: mathematics is a difficult science, incomprehensible to most people, without a history and consequently without any originality. For these reasons, a large number of people have not the vaguest idea about how the many mathematicians in the world spend their working days.

One might be tempted to come to the conclusion that, in any case, it is useless to try to tell people what mathematics is or what today's mathematicians do, as André Weil has stated: "The particular feature of mathematics is that it is not understood by non-mathematicians". Alternatively, one might try to convey the idea of mathematics that engineers have. They are always striving to attain optimal value for their projects and, as Dieudonné comments,[4] "they see mathematics as a rich deposit of formulas, available on request." In other words, mathematics is seen as a sort of container to be dipped into, without the pedantic precision of mathematicians who tend to purposely complicate things that are in themselves very simple.

But, as Musil points out:[5] "if it is comprehensible for an engineer to concentrate wholly on his specialization instead of moving freely in the vast untrammelled spaces of thought....he is not asked to transfer to his private soul, the audacious and innovatory spirit of his technical skill", this is not true for mathematics in which "we find the new logic and the spirit of their essence" in the words of the author of "The Man Without Quality".

If these ideas – the new logic and the spirit of their essence – are to be found in mathematics, then one can understand why mathematicians feel that it is worthwhile to make themselves understood. Dieudonné is one of those who feel that we should at least attempt to find out the reason why mathematics is so little understood, even though the task is difficult, if not actually impossible.

It is clear that the problem of today's mathematics being unintelligible to those who are not mathematicians, makes it even more difficult to see what cultural role mathematics might play. As an example of how wide-open the question is, we should mention the recent debate between mathematicians and non-mathematicians regarding artistic and scientific creativity.

**The Role of Creativity**

*The role of mathematics is both*
*eternal and immortal*
Tommaso Campanella (1568-1639)

One of the main questions is whether the creativity of a mathematician leads him to invent new worlds, or whether he merely discovers a world that already exists. This might seem to be a rather pointless question, but though it may appear to be of little interest to non-mathematicians, this is not the case for mathematicians such as Roger Penrose who has devoted part of his book "The Emperor's New Mind" to this theme[6]. "In mathematics, should we speak of invention or discovery?" he asks.

There are two possible answers to this question: either, when mathematicians attain new results leading to elaborate mental constructions which (even though not having any connection with reality) still possess sufficient strength and elegance of their own to convince the reseacher that such "mere mental constructions" have their own reality; or, do mathematicians discover that these "mere mental constructions are already there" - a truth which is totally independent of their elaboration? Penrose tends toward the latter view although he does point out that the question is not as simple as it seems. His opinion is that, in mathematics, situations arise in which the term *discovery* is certainly more appropriate than *invention*. There are cases, for instance, where the results derive basically from the structure itself, rather than from the mathematician's efforts. Penrose cites complex numbers as an example[7]:

"We find many magical properties that these complex numbers possess, properties that we had no inkling about at first. These properties are just *there.* They were not put there by Cardano, nor by Bombelli, nor Wallis, nor Coates, nor Euler, nor Wessel, nor Gauss, despite the undoubted farsightedness of these, and other, great mathematicians; such magic was inherent in the very structure that they gradually uncovered."

When mathematicians discover a structure of this type, it means they have encountered what Penrose calls a *work of God*. So, are mathematicians just explorers? Fortunately, not all mathematical structures are so strictly predetermined. There are cases in which "the results are derived in equal measure from the structure itself and the mathematician's calculations". In such cases it is more appropriate in Penrose's opinion to use the word *invention* rather than *discovery*. As a consequence, there is space for what Penrose calls *works of man*, but at the same time he goes on to say that far more importance should be attached to the discovery of structures that can be considered *works of God* rather than the invention of mankind.

Similar distinctions can be made in the arts and engineering. Masterpieces of art are in fact *closer to God.* Penrose mantains that, amongst artists, a fairly widely held view is that their more significant works show *eternal truths* which have *a priori existence*, while minor works have a more personal character, and are more arbitrary, human constructions. In mathematics, this need to believe in a disembodied and eternal existence (at least, for the more profound mathematical concepts – the *works of God*) is even more deeply felt. Penrose comments:[8]

"There is a compelling uniqueness and universality in such mathematical ideas which seems to be of quite a different order from that which one could expect in the arts or engineering."

A work of art can be appreciated or criticized in different ages, but no-one can cast doubt on a proper demonstration of a mathematical result. The notion that mathematical ideas have a non-material and timeless existence was first put forward about 360 B.C. by the Greek philosopher Plato. Since then, the term Platonic mathematics has been used. Penrose is explicit when he states that mathematicians think of their discipline as a highly creative activity, no less so than the creativity of artists. In fact, because of the uniqueness and universal nature of mathematical creation, it can be said to be superior to artistic creativity. Penrose does not say so directly but many mathematicians feel that mathematics is the true art. A difficult and laborious art, with its own language and symbolism, producing results that are universally accepted.

A new hero has made its appearance in the last years much increasing the creative possibilities of mathemacians and so amplifying their possible cultural influences.

**The Role of Computer Graphics**

In recent years, we have seen the rise of a relatively new phenomenon, the spread of computers with high graphic capability. This has led to an enormous increase in the quantity of mathematical images which have become very popular even beyond the scientific community.

It is important to note that the use of computer graphics techniques by mathematicians has been used not only just to visualize already known phenomena but in a more interesting way to understand how to solve problems not completely solved. In some specific cases, such techniques have provided a new way of proving results in mathematical research. The computer has become an instrument that allows experiments to be made in mathematics in a sense and a dimension that is wholly new.

1979 is the year of publication of the paper by R. Hersh *Some Proposals for Reviving the Philosophy of Mathematics*[9] and then, two years later, of the volume by P. J. Davis and R. Hersh *The Mathematical Experience*[10]. One of the chapters of the volume by Davis and Hersh is entitled:[11] *Why Should I Believe a Computer?* The two authors recalled the rare event which took place in 1976: an announcement of the proof of a theorem in pure mathematics broke into the news columns of *The New*

*York Times.* The occasion was the proof by K. Appel and W. Haken[12] of the Four-Color Conjecture:

"The occasion was newsworthy for two reasons. To begin with, the problem in question was a famous one… But the method of proof in itself was newsworthy. For an essential part of the proof consisted of computer calculations. That is to say, the published proof contained computer programs and the output resulting from calculations according to the programs. The intermediate steps by which the programs were executed were of course not published; in this sense the published proofs were permanently and in principle incomplete." Davis and Hersh pointed out that "in applied mathematics, the computer serves to calculate an approximate answer, when theory is unable to give us an exact answer… But in no way the theory depends on the computer for its conclusions; rather, the two methods, theoretical and mechanical, are like two independent views of the same object; the problem is to coordinate them… The rigorous mathematics of proof remains uncontaminated by the machine… In the Haken-Appel four-color theorem, the situation is totally different. They present their work as a definitive, complete, rigorous proof. In an expository article on their work, Appel and Haken wrote that most mathematicians who were educated prior to the development of fast computers tend not to think of the computer as a routine tool to be used in conjunction with other and more theoretical tools in advancing mathematical knowledge."

Davis and Hersh were thinking of high-speed computers, the possibility for the machine to make thousands of calculations in a short time. Thomas Banchoff and Charles Strauss at Brown University at the end of the 1970s had the idea of using computer graphics animation to visually investigate the geometrical and topological properties of three-dimensional surfaces. This approach to the use of computers in mathematical research was new. It became possible to construct a surface on the video terminal and move it and transform it in order to better understand its properties. It has become a new way of constructing models as well as a good help for intuition.

Computer graphics works not only as a pure visualization of almost well-known phenomena but also as a new way of studying mathematical problems, in particular geometrical ones. It can be said that a new branch of mathematics has been developing in the last few years that can be called *Visual Mathematics*[13]. The great potential of computer graphics as a new exploratory medium was recognized by mathematicians soon after the relevant technology became available. As display devices and programming methods grew more sophisticated, so did the depth and scope of applications of computer graphics to mathematical problems. In some cases, the use of computer graphics was essential to obtain a formal proof of the existence of the new surfaces[14].

The possibilities offered by the new visual techniques have opened new horizons to the mathematicians and have widened the diffusion and so the influence that their ideas can have not only on the other branches of sciences but on culture at large. An important role in this area can be played by exhibitions and museums dedicated to the visual aspects of contemporary mathematics, including the possible relationships to their artistic appeal.

**A Possible Approach: Exhibitions and Museums of Art and Mathematics**

During 1989, Italy was host to a number of exhibitions touching on the theme of art and science with particular reference to Art and Mathematics. The most important occasion was the organization of the exhibition *L'occhio di Horus: itinerari nell'immaginario matematico* (The Eye of Horus: Itineraries in the Mathematical Imagination). It was organized in cooperation with the Istituto della Enciclopedia Italiana, the Istituto Italiano di Studi Filosofici of Naples and the Musée des Sciences et de l'Industrie, Parc de la Villette, Paris. The idea of the exhibition and its scientific and artistic coordination was handled by a mathematician.

The idea behind the exhibition was to make a contribution towards spreading what can be called the *Culture of Mathematics* as widely as possible. It was to be an exhibition dominated by the visual aspect, by images. Knowing that it is important to make a few basic mathematical concepts interesting and entertaining for the public (and I say a few because I am convinced that there are many subjects that cannot be presented for non-experts) it was attempted to make these ideas visible by enlisting the aid not only of mathematicians but also of artists who might be interested in such themes through their work.

The reason was to reinforce the idea that this was not a purely didactic exhibition on mathematics, but rather a real cultural exhibition which contained several more strictly didactic sections. The other aspect, the informative and entertaining side, was partly covered by the *Horizons Mathématiques* sector from La Villette, a display that was included in the larger exhibition *The Eye of Horus*.

The exhibition was arranged in thematic sectors, each of which contained explanatory panels, one or two interractive tables from La Villette, some films (most from the series *Art and Mathematics*[15], and works by Italian and foreign artists including paintings, sculptures and computer graphics works.

The first section had the same title as the exhibition itself: *The Eye of Horus*. The image of the exhibition was a painting by Fabrizio Clerici on the theme of Horus' eye.

The reason for choosing this subject was that the eye of Horus had a precise mathematical significance for the Egyptians, closely linked to the myth of Isis and Osiris, well-known in ancient times and handed down to us by Plutarch, amongst others. The other sections were devoted to numbers and to brief profiles of all the mathematicians mentioned in the display panels.

Then symmetry, where there had to be a section on the Dutch artist M. C. Escher[16]; one area dealt with non-periodic tilings, the so-called Penrose tiles, and the quasi-crystals.

The following section, dealing with soap bubbles (or minimal surfaces), presented a brief artistic and mathematical history of the various shapes that can be obtained with soap films, and more recently by means of computer graphics[17].

A large area was given over to the Platonic Solids, with sculptures and paintings by contemporary artists[18]. Amongst the exhibits, there was a much-enlarged model of the Mazzocchio, the head-covering which appeared in so many of Paolo Uccello's paintings during the Italian Renaissance.

Topology occupied a large area divided into subsectors, devoted to knots, the Moebius strip[19], and labyrinths. The labyrinth room contained reconstructions in scale of several famous labyrinths (Crete, garden maze of Villa Pisani) and some large paintings by the painter Clerici.

One room was given over to the Etruscan Venus, the computer image obtained by artists, mathematicians and computer experts working together at the University of Illinois at Urbana[20].

The last section dealt with the fourth dimension. In addition to large steel sculptures of four-dimensional solids (by the Italian artist Attilio Pierelli), visitors could actually go inside a sculpture entitled *Truncated 600-cell*, created by Harriet Brisson, which gave the impression of travelling in space of higher dimensions!

Alongside the exhibition there was a book written by mathematicians and artists and divided into the same sections as the exhibition. But it was much more that just a catalogue of the exhibition. It was a book in every sense, with articles written for the occasion, a book that can be used independently of the exhibition. Of course is was also highly illustrated[21]. In addition, Italian television made a documentary of the show in the series "The Great Exhibition of the Year" – a real cultural event based on contemporary mathematics and computer graphics![22]

**Final Remarks**

I have tried to discuss which possible role mathematical research could have in contemporary culture and how the importance of this role is increasing together with a larger diffusion of computer tools with high graphics facilities. Mathematicians are becoming aware of the increasing importance they could have not only in relationships with the other scientific disciplines but with all the various aspects of cultural development. After many years mathematicians are becoming a community with a recognizable identity even for the general public.

It can happen that a very specific fact like the proof of a theorem, a well-known theorem like "The Last Fermat's Theorem", whose terms are easily understood, becomes a cultural event of which all newspapers in the world are obliged to write. A even more surprising fact as it is almost impossible to understand for the non-specialists, that is, the mathematicians working in this specific area, the details of the proof. It became important to know that the Fermat's last theorem has been proved. The interest for what mathematicians are doing is really changing in the last years. A recent paper was published in the Notices of the American Mathematical Society by the title[23] "Fermat Fest Draws a Crowd: 1000 People Attend an Evening of Public Lectures Sponsored by MSRI" (MSRI, Mathematical Sciences Research Institute in Berkeley, CA). According to William P. Thurston, director of the MSRI, the mathematical community is often skeptical that the public has any interest in mathematics. "The Fermat Fest shows that there are lots of people who will like mathematics if it's presented well...The Fermat Fest was appealing because of the human element and because it's just interesting. It is not beacuse the mathematics was disguised as something else."

New horizons open up for mathematicians; perhaps people will finally agree that the French mathematician Le Lionnais was wright when he wrote[24]:

"S'il existe encore des pseudo-humanistes pour qui l'incompréhension des mathématiques constitue un titre de gloire, le nombre croissant des profanes qui regrettent de ne pouvoir participer pleinement à ce banquet des Dieux et souhaitent bénéficier au moins des miettes du festin est plutôt rassurant."

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